EARTHQUAKE ENGINEERING AND STRUCTURAL DYNAMICS
Earthquake Engng. Struct. Dyn. **27**, 1141–1147 (1998)

BENCHMARK PROBLEMS IN STRUCTURAL CONTROL: PART II—ACTIVE TENDON SYSTEM

B. F. SPENCER JR.,[1,*,†] S. J. DYKE[2,‡] AND H. S. DEOSKAR[1,§]
[1]*Department of Civil Engineering and Geological Science, University of Notre Dame, Notre Dame IN 46556-0767, U.S.A.*
[2]*Department of Civil Engineering, Washington University, St. Louis, MO 63130-4899, U.S.A.*

SUMMARY

In a companion paper (Spencer *et al.*[6]), an overview and problem definition was presented for a well-defined benchmark structural control problem for a model building configured with an Active Mass Driver (AMD). A second benchmark problem is posed here based on a high-fidelity analytical model of a three-storey, tendon-controlled structure at the National Center for Earthquake Engineering Research (NCEER). The purpose of formulating this problem is to provide another setting in which to evaluate the relative effectiveness and implementability of various structural control algorithms. To achieve a high level of realism, an *evaluation* model is presented in the problem definition which is derived directly from experimental data obtained for the structure. This model accurately represents the behaviour of the laboratory structure and fully incorporates actuator/sensor dynamics. As in the companion paper, the evaluation model will be considered as the real structural system. In general, controllers that are successfully implemented on the evaluation model can be expected to perform similarly in the laboratory setting. Several evaluation criteria are given, along with the associated control design constraints. © 1998 John Wiley & Sons, Ltd.

KEY WORDS: structural control; benchmark problem; earthquake; active tendon control

EXPERIMENTAL STRUCTURE

The structure on which the evaluation model is based is the actively controlled, three-storey, single-bay, model building considered in Chung *et al.*[1] The test structure, shown in Figures 1 and 2, has a mass of 2950 kg, distributed among the three floors, and is 254 cm in height. The ratio of model quantities to those corresponding to the prototype structure are: force = 1 : 16, mass = 1 : 16, time = 1 : 2, displacement = 1 : 4 and acceleration = 1 : 1. Due to the time scaling, the natural frequencies of the model are approximately twice those of the prototype. The first three modes of the model system are at 2·27, 7·33, and 12·24 Hz, with associated damping ratios given, respectively, by 0·6, 0·7, and 0·3 per cent.

A hydraulic control actuator, four pretensioned tendons, and a stiff steel frame connecting the actuator to the tendons are provided to apply control forces to the test structure. The four diagonal tendons transmit the force from the control actuator to the first floor of the structure, and the steel frame connects the actuator to the tendons. Because hydraulic actuators are inherently open-loop unstable, a feedback control system is

* Correspondence to: B. F. Spencer, Jr., Department of Civil Engineering and Geological Science, University of Notre Dame, Notre Dame, IN 46556-0767, U.S.A.
† Professor
‡ Assistant Professor
§ Former Graduate Assistant

Contract/grant sponsor: National Science Foundation; Contract/grant numbers: CMS 95-00301 and CMS 95-28083

CCC 0098–8847/98/111141–07$17·50

Received 10 August 1997
Revised 2 December 1997

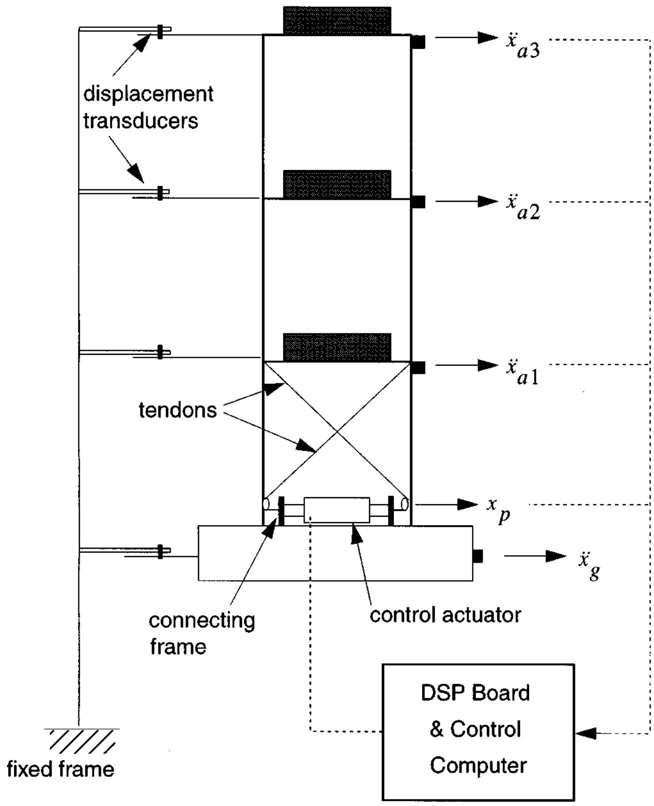

Figure 1. Schematic of experimental setup

employed to stabilize the control actuator and improve its performance. This feedback signal is a combination of the position, velocity and pressure measurements. For this actuator, an Linear Variable Differential Transformer (LDVT), rigidly mounted to the piston, provides the displacement measurement, which is the primary feedback signal. This measurement is also sent through an analog differentiator to provide a velocity measurement, and a pressure transducer across the actuator piston provides the pressure measurement.

The structure was fully instrumented to provide for a complete record of the motions undergone by the structure during testing. Accelerometers positioned on each floor of the structure measured the absolute accelerations, and an accelerometer located on the base measured the ground excitation, as shown in Figure 1. The displacement of the actuator was measured using the LVDT mentioned above. Force transducers were placed in series with each of the four tendons and their individual outputs were combined to determine the total force applied to the structure. Additional measurements were also available for model verification. Displacement transducers on the base and on each floor were attached to a fixed frame (i.e. not attached to the earthquake simulator), as shown in Figure 1, to measure the absolute displacements of the structure and of the base. The relative displacements were determined by subtracting the base displacement from the absolute displacement of each floor. Because a fixed frame was necessary to measure the relative displacements of the structure, these measurements would not be directly available in a full-scale implementation. Thus, the displacement measurements are used only for model verification and are not available for feedback in the control system.

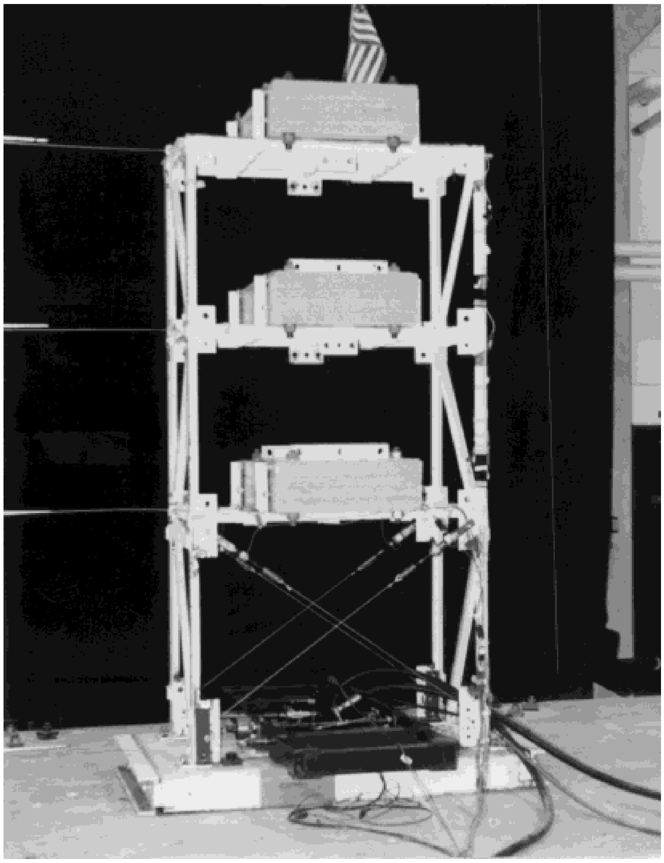

Figure 2. Three-degree-of-freedom test structure

EVALUATION MODEL

A high-fidelity, linear time-invariant state-space representation of the input-output model for the structure described in the previous section has been developed. The model has 20 states and is of the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{E}\ddot{x}_{\mathrm{g}} \tag{1}$$

$$\mathbf{y} = \mathbf{C}_y\mathbf{x} + \mathbf{D}_y u + \mathbf{F}_y\ddot{x}_{\mathrm{g}} + \mathbf{v} \tag{2}$$

$$\mathbf{z} = \mathbf{C}_z\mathbf{x} + \mathbf{D}_z u + \mathbf{F}_z\ddot{x}_{\mathrm{g}} \tag{3}$$

where $\mathbf{x}$ is the state vector, $\ddot{x}_{\mathrm{g}}$ is the scalar ground acceleration, u is the scalar control input, $\mathbf{y} = [x_{\mathrm{p}}, \ddot{x}_{\mathrm{a}1}, \ddot{x}_{\mathrm{a}2}, \ddot{x}_{\mathrm{a}3}, f, \ddot{x}_{\mathrm{g}}]'$ is the vector of responses that can be directly measured, $\mathbf{z} = [x_1, x_2, x_3, x_{\mathrm{p}}, \dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_{\mathrm{p}}, \ddot{x}_{\mathrm{a}1}, \ddot{x}_{\mathrm{a}2}, \ddot{x}_{\mathrm{a}3}, f]'$ is the vector of responses that can be regulated. Here, x_i is the displacement of the ith floor relative to the ground, x_{p} is the displacement of the control actuator, $\ddot{x}_{\mathrm{a}i}$ is the absolute acceleration of the ith floor, $\mathbf{v}$ is the vector of measurement noises, and $\mathbf{A}$, $\mathbf{B}$, $\mathbf{E}$, $\mathbf{C}_y$, $\mathbf{D}_y$, $\mathbf{C}_z$, $\mathbf{D}_z$, $\mathbf{F}_y$ and $\mathbf{F}_z$ are matrices of appropriate dimension. The coefficient matrices in eq. (1)–(3) are determined from the data collected at the NCEER using the identification methods presented in Dyke *et al.*[2–5] The resulting model represents the input–output behaviour of the structural system up to 50 Hz and includes the effects of

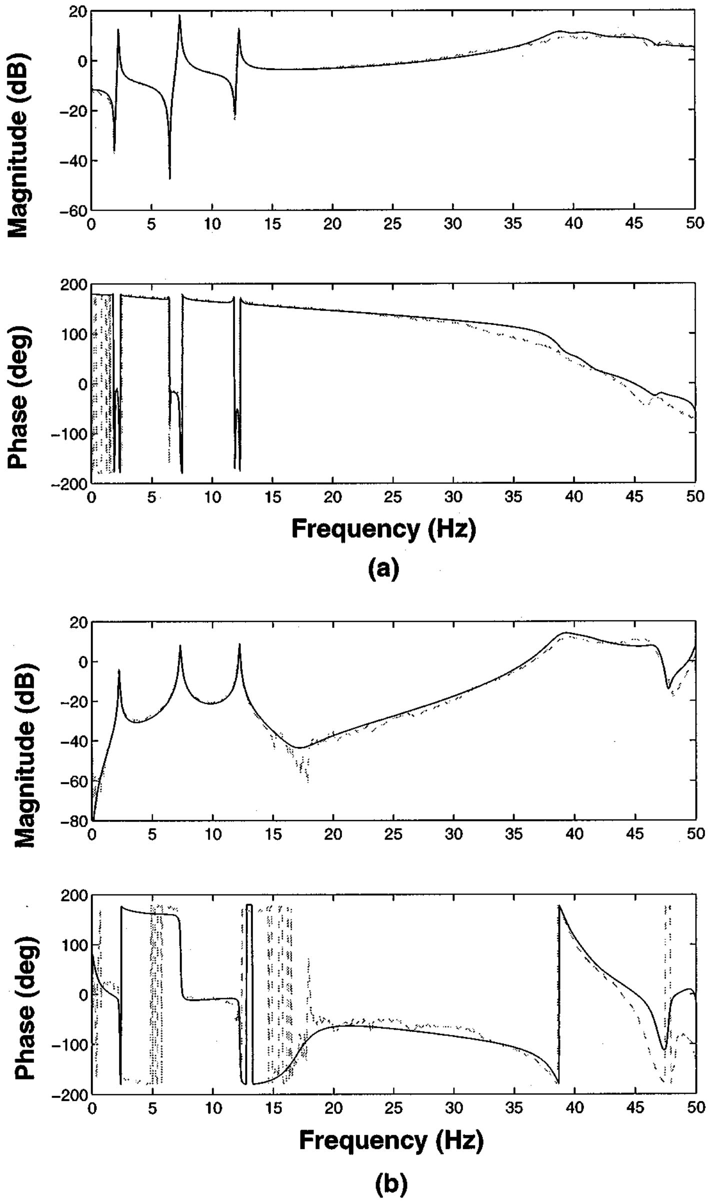

Figure 3. Representative comparison of the transfer functions for the test structure and the evaluation model: (a) actuator command to the force in the tendons; (b) actuator command to the third floor absolute acceleration

actuator/sensor dynamics and control-structure interaction. Figure 3 provides a representative comparison between selected model transfer functions and the experimental data.

The model given in eq. (1)–(3) is termed the *evaluation* model and will be used to assess the performance of candidate controllers; that is, the evaluation model is considered herein to be the true representation of the structural system.

CONTROL DESIGN PROBLEM

The control design problem stated here follows the companion paper[6] and is stated as: determine a discrete-time, feedback compensator of the form

$$\mathbf{x}^{c}_{k+1} = f_1(\mathbf{x}^{c}_k, \underset{\sim}{\mathbf{y}}_k, u_k, k) \tag{4}$$

$$u_k = f_2(\mathbf{x}^{c}_k, \underset{\sim}{\mathbf{y}}_k, k) \tag{5}$$

where $\mathbf{x}^{c}_k$, $\underset{\sim}{\mathbf{y}}_k$ and u_k are the state vector for the compensator, the output vector and the control command, respectively, at time $t = kT$. For this problem, $\dim(\mathbf{x}^{c}) \leqslant 12$ is required, and the performance of all control designs must be assessed using the evaluation model described previously. For each proposed control design, performance and stability robustness should be discussed. As detailed in the following paragraphs, the merit of a controller will be based on criteria given in terms of both rms and peak response quantities. Normally, smaller values of the evaluation criteria indicate superior performance.

Evaluation criteria: *RMS responses*

Assume that the input excitation $\ddot{x}_g$ is a stationary random process with a spectral density defined by the Kanai–Tajimi spectrum

$$S_{\ddot{x}_g\ddot{x}_g}(\omega) = \frac{S_0(4\zeta_g^2\omega_g^2\omega^2 + \omega_g^4)}{(\omega^2 - \omega_g^2)^2 + 4\zeta_g^2\omega_g^2\omega^2} \tag{6}$$

where ω_g and ζ_g are unknown, but assumed to lie in the following ranges: $8\ \text{rad/sec} \leqslant \omega_g \leqslant 50\ \text{rad/sec}$, $0{\cdot}3 \leqslant \zeta_g \leqslant 0{\cdot}75$. To have a basis for comparison, the spectral intensity is chosen such that the rms value of the ground motion takes a constant value of $\sigma_{\ddot{x}_g} = 3{\cdot}4 \times 10^{-2}$g's, i.e.

$$S_0 = \frac{2{\cdot}35 \times 10^{-3}\,\zeta_g}{\pi\omega_g(4\zeta_g^2 + 1)}\ \text{g}^2\ \text{sec} \tag{7}$$

The first criterion on which controllers will be evaluated is based on their ability to minimize the maximum rms interstorey drift due to all admissible ground motions. Therefore, the non-dimensionalized measure of performance is given by

$$J_1 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_{d_1}}{\sigma_{x_{3o}}}, \frac{\sigma_{d_2}}{\sigma_{x_{3o}}}, \frac{\sigma_{d_3}}{\sigma_{x_{3o}}} \right\} \tag{8}$$

where σ_{d_i} is the stationary rms interstorey drift for the ith floor, and $\sigma_{x_{3o}} = 2{\cdot}34$ cm is the worst-case stationary rms displacement of the third floor of the uncontrolled building over the class of excitations considered (occurring when $\omega_g = 14{\cdot}5$ rad/sec, $\zeta_g = 0{\cdot}3$). The inter-storey drifts are given, respectively, by $d_1(t) = x_1(t)$, $d_2(t) = x_2(t) - x_1(t)$ and $d_3(t) = x_3(t) - x_2(t)$.

A second evaluation criterion is given in terms of the maximum rms absolute acceleration, yielding a performance measure given by

$$J_2 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_{\ddot{x}_{a1}}}{\sigma_{\ddot{x}_{a3o}}}, \frac{\sigma_{\ddot{x}_{a2}}}{\sigma_{\ddot{x}_{a3o}}}, \frac{\sigma_{\ddot{x}_{a3}}}{\sigma_{\ddot{x}_{a3o}}} \right\} \tag{9}$$

where $\sigma_{\ddot{x}_{ai}}$ is the stationary rms acceleration for the ith floor, and $\sigma_{\ddot{x}_{a3o}} = 0{\cdot}485$g's is the worst-case stationary rms acceleration of the third floor of the uncontrolled building (occurring when $\omega_g = 14{\cdot}5$ rad/sec, $\zeta_g = 0{\cdot}3$).

The hard constraints for the control effort are given by $\sigma_u \leqslant 1$ volt, $\sigma_f \leqslant 4$ KN and $\sigma_{x_p} \leqslant 1$ cm. Additionally, candidate controllers are to be evaluated based on the required control resources. Three quantities, σ_{x_p}, $\sigma_{\dot{x}_p}$ and σ_f, should be examined to make the assessment. The rms actuator displacement, σ_{x_p}, provides a measure of the required physical size of the device. The rms actuator velocity, $\sigma_{\dot{x}_p}$, provides a measure of the control power required. The rms absolute acceleration σ_f provides a measure of the magnitude of the forces that the actuator must generate to execute the commanded control action. Therefore, the

nondimensionalized control resource evaluation criteria are

$$J_3 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_{x_p}}{\sigma_{x_{3o}}} \right\} \tag{10}$$

$$J_4 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_{\dot{x}_p}}{\sigma_{\dot{x}_{3o}}} \right\} \tag{11}$$

$$J_5 = \max_{\omega_g, \zeta_g} \left\{ \frac{\sigma_f}{W} \right\} \tag{12}$$

where $\sigma_{\dot{x}_{3o}} = 33{\cdot}3$ cm/sec is the worst-case stationary rms velocity of the third floor relative to the ground for the uncontrolled structure (occurring when $\omega_g = 14{\cdot}5$ rad/sec, $\zeta_g = 0{\cdot}3$), and W is the weight of the building (289 kN).

Evaluation criteria: *peak responses*

Here, the input excitation $\ddot{x}_g$ is assumed to be a historical earthquake record. Both the 1940 El Centro NS record and the NS record for the 1968 Hachinohe earthquake are considered. Because the system under consideration is a scale model, the time scale should be increased by a factor of 2 (i.e. the earthquakes occur in 1/2 the recorded time). The required scaling of the magnitude of the ground acceleration is 1. The evaluation criterion is based on minimization of the non-dimensionalized peak interstorey drifts due to both earthquake records. For each earthquake, the maximum drifts are non-dimensionalized with respect to the uncontrolled peak third floor displacement, denoted x_{3o}, relative to the ground. Therefore, the performance measure is given by

$$J_6 = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \left\{ \frac{|d_1(t)|}{x_{3o}}, \frac{|d_2(t)|}{x_{3o}}, \frac{|d_3(t)|}{x_{3o}} \right\} \right] \tag{13}$$

A second performance evaluation criterion is given in terms of the peak acceleration, yielding

$$J_7 = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \left\{ \frac{|\ddot{x}_{a1}(t)|}{\ddot{x}_{a3o}}, \frac{|\ddot{x}_{a2}(t)|}{\ddot{x}_{a3o}}, \frac{|\ddot{x}_{a3}(t)|}{\ddot{x}_{a3o}} \right\} \right] \tag{14}$$

where the accelerations are nondimensionalized by the peak uncontrolled third floor acceleration, denoted $\ddot{x}_{a3o}$, corresponding, respectively, to each earthquake.

The control constraints are $\max_t |u(t)| \leqslant 3$ volts, $\max_t |x_p(t)| \leqslant 3$ cm, $\max_t |f(t)| \leqslant 12$ kN, and both the El Centro and the Hachinohe earthquakes should again be considered. Additionally, the candidate controllers are to be evaluated in terms of the required control resources as follows:

$$J_8 = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \frac{|x_p(t)|}{x_{3o}} \right] \tag{15}$$

$$J_9 = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \frac{|\dot{x}_p(t)}{\dot{x}_{3o}} \right] \tag{16}$$

$$J_{10} = \max_{\substack{\text{El Centro} \\ \text{Hachinohe}}} \left[\max_t \frac{|f(t)}{W} \right] \tag{17}$$

where $\dot{x}_{3o}$ is the peak uncontrolled third floor relative velocity corresponding, respectively, to each earthquake.

For the half-scale El Centro earthquake, $x_{3o} = 6{\cdot}45$ cm, $\dot{x}_{3o} = 99{\cdot}9$ cm/sec and $\ddot{x}_{a3o} = 1{\cdot}57g$'s. For the half-scale Hachinohe earthquake, $x_{3o} = 36{\cdot}78$ cm, $\dot{x}_{3o} = 56{\cdot}1$ cm/sec and $\ddot{x}_{a3o} = 0{\cdot}778g$'s.

CONTROL IMPLEMENTATION CONSTRAINTS

With the following exceptions, the implementation constraints are identical to those specified in the companion paper.[6] The measurements that are directly available for use in determination of the control action are $\mathbf{y} = [x_p, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, f, \ddot{x}_g]'$. The psuedo-velocities, $\underset{\sim}{\dot{x}}_{a1}, \underset{\sim}{\dot{x}}_{a2}, \underset{\sim}{\dot{x}}_{a3}, \underset{\sim}{\dot{x}}_g$, are also available for determination of the control action, resulting in a combined output vector given by $\underset{\sim}{\mathbf{y}} = [x_p, \ddot{x}_{a1}, \ddot{x}_{a2}, \ddot{x}_{a3}, f, \ddot{x}_g, \underset{\sim}{\dot{x}}_{a1}, \underset{\sim}{\dot{x}}_{a2}, \underset{\sim}{\dot{x}}_{a3}, \underset{\sim}{\dot{x}}_g]'$.

CLOSURE

The 20-state evaluation model, as well as the 10-state control design model, the input data and the simulation model are available on the World Wide Web at the following URL:

http://www.nd.edu/~quake/

Additionally, all control designs reported in this special issue of the Journal are available at this URL. If you cannot access the World Wide Web or have questions regarding the benchmark problem, please contact the senior author via e-mail: spencer@nd.edu

ACKNOWLEDGMENTS

This research is partially supported by National Science Foundation Grant Nos. CMS 95-00301 and CMS 95-28083 (Dr S. C. Liu, Program Director). The input provided by the Committee on Structural Control, ASCE Structural Division is also acknowledged. The assistance of Dr Erik A. Johnson in editing the control designs, and preparing the Web site results is gratefully appreciated.

APPENDIX

Nomenclature (supplement to companion paper)

x_p	displacement of the actuator piston relative to the ground
$\dot{x}_p$	velocity of the actuator piston relative to the ground
f	net force in the tendons
σ_{x_p}	rms displacement of the actuator piston relative to the ground
$\sigma_{\dot{x}_p}$	rms velocity of the actuator piston relative to the ground
σ_f	rms force in the tendons

REFERENCES

1. L. L. Chung, R. C. Lin, T. T. Soong and A. M. Reinhorn, 'Experiments on active control for MDOF seismic structures', *J. Engng. Mech. ASCE* **115**, 1609–1627 (1989).
2. S. J. Dyke, B. F. Spencer Jr., A. E. Belknap, K. J. Ferrell, P. Quast and M. K. Sain, 'Absolute acceleration feedback control strategies for the active mass driver', *Proc. 1st World Conf. on Structural Control*, Vol. 2, Pasadena, California, 3–5 August 1994.
3. S. J. Dyke, B. F. Spencer Jr., P. Quast, M. K. Sain, D. C. Kaspari Jr. and T. T. Soong, 'Experimental verification of acceleration feedback control strategies for an active tendon system', *National Center for Earthquake Engineering Research Tehnical Report NCEER-94-0024*, 29 August 1994.
4. S. J. Dyke, B. F. Spencer Jr., P. Quast, D. C. Kaspari Jr. and M. K. Sain, 'Implementation of an active mass driver using acceleration feedback control', *Microcomputers in Civil Engng.* **11**, 305–323 (1996).
5. S. J. Dyke, B. F. Spencer Jr., P. Quast, M. K. Sain, D. C. Kaspari Jr. and T. T. Soong, 'Acceleration feedback control of MDOF structures', *J. Engng. Mech. ASCE* **122**(9), 907–918 (1996).
6. B. F. Spencer Jr., S. J. Dyke and H. S. Deoskar, 'Benchmark problems in structural control: Part I—active mass driver system', *Earthquake Engng. Struct. Dyn.* **27**(11), 1127–1139 (1998).
7. MATLAB The Math Works, Inc. Natick, MA, 1994.

